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Thermodynamics of 2D string theory

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Abstract: We calculate the free energy, energy and entropy in the matrix quantum mechanical formulation of 2D string theory in a background strongly perturbed by tachyons with the imaginary minkowskian momentum $\pm i/R$ ("Sine-Liouville" theory). The system shows a thermodynamical behaviour corresponding to the temperature $T = 1/(2\pi R)$. We show that the microscopically calculated energy of the system satisfies the usual thermodynamical relations and leads to a non-zero entropy.

Keywords: Bosonic Strings, Matrix Models.

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Contents

1. Introduction

Nontrivial backgrounds in the string theory often have a thermodynamical behaviour corresponding to a certain temperature and a classically big entropy. Typical examples are the black holes where the thermodynamics manifests itself by the Hawking radiation at a certain temperature T_H and the Bekenstein-Hawking entropy. The latter is expressed through the classical parameters of the system (the area of the horizon) and satisfies the standard thermodynamical relations with the free energy and energy (mass of the black hole).[1]

In this paper we will try to demonstrate that the thermodynamical behaviour with a temperature T is a rather natural phenomenon in the 2-dimensional string theory in specific backgrounds created by tachyon sources with euclidean momenta corresponding to the Matsubara frequencies $2\pi k/T, k = 1, 2, \ldots$. For $k = 1$ this system is T-dual to the so called "Sine-Liouville" theory conjectured to describe 2D string theory on the euclidean black hole background [3]. The latter can be obtained introducing a vortex source with the euclidean time compactified on a radius $R = 1/2\pi T$. It was studied in [4] and further in [5, 6] in the matrix quantum mechanical (MQM) formulation using the integrability properties.

We will use here the MQM in the singlet sector to study this system in minkowskian time, perturbed by a source of "tachyons" with imaginary momenta. This approach, elaborated in the paper [7], using some early ideas of [8], extensively relies on the classical Toda

[1] For review of black hole physics see [1, 2].

integrability and the representation of the system in terms of free fermions arising from the eigenvalues of the matrix field. We will demonstrate that the microscopically defined energy of the system, calculated as the energy of (perturbed) Fermi sea, coincides with its thermodynamical counterpart E calculated as the derivative of the free energy with respect to the temperature. The free energy itself can be found from the Fermi sea due to its relation with the number of particles $N = \partial\mathcal{F}/\partial\mu$.

This coincidence of the microscopic and thermodynamical energies looks satisfactory, though a little bit mysterious to us. We could not figure out here where the entropy, also following from this derivation, comes from since the system looks like a single classical state of the moving Fermi liquid. The formulas become especially simple in the black hole limit proposed in [4], where the energy and entropy dominate with respect to the free energy. On the other hand, the entropy naturally disappears in the opposite limit of the trivial linear dilaton background where the perturbation source is absent.

The entropy seems to be more visible in the T-dual formulation of the theory based on the vortex perturbation in the compact euclidean time where we deal with the higher representations of the SU(N) symmetry of the MQM [9]. However the direct counting of the nonsinglet states appears to be technically a difficult problem. But we still think that our observation could shed light on the origin of the entropy of the black hole type solutions in the string theory.

The paper is organized as follows. In the next section we review the fermionic representation of $c = 1$ string theory. In section 3 we show how one can describe the tachyon perturbations in this formalism. Section 4 presents the explicit solution for the particular case of the Sine-Liouville theory. In section 5 we evaluate the grand canonical free energy and energy of the system using the techniques of integration over the Fermi sea. In section 6 we demonstrate that the quantities found in the previous section satisfy the thermodynamical relations and give rise to a nonvanishing entropy. The parameter R of the perturbations can be interpreted as the inverse temperature. Finally, we discuss our results and remaining problems.

2. The $c = 1$ string theory as the collective theory of the Fermi sea

The $c = 1$ string theory can be formulated in terms of collective excitations of the Fermi sea of the upside-down harmonic oscillator [10, 11, 12, 13]. The tree-level tachyon dynamics of the string theory is contained in the semiclassical limit of the ensemble of fermions, in which the fermionic density in the phase space is either one or zero. Each state of the string theory corresponds to a particular configuration of the incompressible Fermi liquid [14, 15, 16].

In the quasi-classical limit the motion of the fermionic liquid is determined by the classical trajectories of its individual particles in the phase space. It is governed by the one-particle hamiltonian

$$H_0 = \frac{1}{2}\left(p^2 - x^2\right), \tag{2.1}$$

where the coordinate of the fermion x stands for an eigenvalue of the matrix field and $p = -i\frac{\partial}{\partial x}$ is its conjugated momentum. We introduce the chiral variables

$$z_\pm(t) = \frac{x(t) \pm p(t)}{\sqrt{2}}\,, \tag{2.2}$$

and define the Poisson bracket as $\{f, g\} = \frac{\partial f}{\partial z_-}\frac{dg}{\partial z_+} - \frac{\partial f}{\partial z_+}\frac{dg}{\partial z_-}$. In these variables $H_0 = -z_+ z_-$ and the equations of motion have a simple solution

$$z_\pm(t) = e^{\pm t} z_\pm\,, \tag{2.3}$$

where the initial values $z_\pm$ parameterize the points of the Fermi sea. Each trajectory represents a hyperbole $H_0(p, x) = E$. The state of the system is completely characterized by the profile of the Fermi sea that is a curve in the phase space which bounds the region filled by fermions. For the ground state, the Fermi sea is made by the classical trajectories with $E < -\mu$, and the profile is given by the hyperbole

$$z_+ z_- = \mu\,. \tag{2.4}$$

The state is stationary since the Fermi surface coincides with one of the classical trajectories and thus the form of the Fermi sea is preserved in time. For an arbitrary state of the Fermi sea, the Fermi surface can be defined more generally by

$$z_+ z_- = M(z_+, z_-)\,. \tag{2.5}$$

It is clear from (2.3) that a generic function M in (2.5) leads to a time-dependent profile. However, this dependence is completely defined by (2.3) and it is of little interest to us. We can always replace (2.5) by the equation for the initial values $z_\pm$.

For a given profile, we define the energy of the system and the number of fermions as the following integrals over the Fermi sea

$$E_0 = \frac{1}{2\pi}\iint\limits_{\text{Fermi sea}} dx\, dp\; H_0(x,p)\,, \qquad N = \frac{1}{2\pi}\iint\limits_{\text{Fermi sea}} dx\, dp\,. \tag{2.6}$$

It is implied that the integrals are bound by a cut-off at a distance $\sqrt{2\Lambda}$. For example, we can restrict the integration to $0 < z_\pm < \sqrt{\Lambda}$. For the ground state (2.4), dropping non-universal terms proportional to the cut-off one reproduces the well known result [17]

$$E_0(\mu) = -\frac{1}{4\pi}\mu^2 \log\left(\frac{\mu}{\Lambda}\right), \qquad N(\mu) = \frac{1}{2\pi}\mu \log\left(\frac{\mu}{\Lambda}\right). \tag{2.7}$$

Given the number of fermions, one can also introduce the grand canonical free energy through the relation

$$N = \frac{\partial \mathcal{F}}{\partial \mu}\,, \tag{2.8}$$

where $\mathcal{F}$ is given in terms of the partition function for a time interval T by

$$\mathcal{F} = -\log \mathcal{Z}/T. \tag{2.9}$$

For the case of the ground state, one finds from (2.7) that the (universal part of the) grand canonical free energy is related to the energy of the fermions as

$$\mathcal{F} = E_0 + \mu N = \frac{1}{4\pi}\mu^2 \log\left(\frac{\mu}{\Lambda}\right). \tag{2.10}$$

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3. The profile of the Fermi sea for time dependent tachyon backgrounds

The ground state (2.4) describes the simplest, linear dilaton background of the bosonic string. Now we would like to study more general backgrounds characterized by condensation of tachyons with nonzero momenta. Such backgrounds correspond to time-dependent profiles (2.5), characterized by the function $M(z_+, z_-)$. We restrict ourselves to the states that contain only tachyon excitations whose momenta belong to a discrete lattice

$$p_n = \frac{in}{R}, \quad n \in \mathbb{Z}. \tag{3.1}$$

They look as Matsubara frequencies corresponding to the temperature $T = 1/(2\pi R)$, or to the compactification in the euclidean time with the radius R. We will perturb the system in the same manner as in [7]. It has been shown that at the quasiclassical level the perturbations can be completely characterized by the asymptotics of the profile of the Fermi sea at $z_+ \gg z_-$ and $z_- \gg z_+$. Accordingly, the equation for the profile can be written in two forms which should be compatible with each other [7]

$$z_+ z_- = M_\pm(z_\pm) = \sum_{k=1}^{n} k t_{\pm k} z_\pm^{k/R} + \mu + \sum_{k=1}^{\infty} v_{\pm k} z_\pm^{-k/R}, \tag{3.2}$$

where $t_{\pm k}$ are coupling constants associated with the perturbing operators and defining the asymptotics of the Fermi sea for $z_+ \gg z_-$ and $z_- \gg z_+$ correspondingly. $v_{\pm k}$ are the coefficients to be found, which completely fix the form of the profile.

It was shown in [7] that such perturbations are described by the dispersionless limit of a constrained Toda hierarchy. The phase space coordinates $z_\pm$ play the role of the Lax operators, $t_{\pm k}$ are the Toda times, and eq. (3.2) appears as the constraint (string equation) of the hierarchy [6].

The integrability allows to find the explicit solution for the profile. It is written in the parametric form as follows

$$z_\pm(\omega, \mu) = e^{-\frac{1}{2R}\chi(\mu)} \omega^{\pm 1} \left(1 + \sum_{k=1}^{n} a_{\pm k}(\mu)\; \omega^{\mp k/R} \right), \tag{3.3}$$

where χ is the so called string susceptibility related to the Toda τ-function as $\chi = \partial_\mu^2 \log \tau$. The latter is the generating function for the coefficients $v_{\pm k}$ in (3.2)

$$v_{\pm k} = \partial_{t_{\pm k}} \log \tau\,, \tag{3.4}$$

which can be identified with one-point vertex correlators [5, 7]. Moreover, the change of variables $z_\pm \to \log \omega, \mu$ turns out to be canonical, i.e.,

$$\{\mu, \log \omega\} = 1\,. \tag{3.5}$$

To find the coefficients $a_{\pm k}$ it is enough to use a simple procedure suggested in [6]. One should substitute the expressions (3.3) in the profile equations (3.2) and compare the coefficients in front of $\omega^{\pm k/R}$.

4. Solution for Sine-Gordon coupled to gravity

Let us restrict ourselves to the case $n = 1$ corresponding to the Sine-Gordon theory coupled to gravity or the so called Sine-Liouville theory. In this case there are only $t_{\pm 1}$ coupling constants and the equation (3.3) takes the form [5]

$$z_{\pm} = e^{-\frac{1}{2R}\chi}\omega^{\pm 1}\left(1 + a_{\pm}\omega^{\mp\frac{1}{R}}\right), \tag{4.1}$$

where χ can be found from [4, 18]

$$\mu e^{\frac{1}{R}\chi} + \left(\frac{1}{R} - 1\right) t_1 t_{-1} e^{\frac{2R-1}{R^2}\chi} = 1\,, \qquad a_{\pm} = t_{\mp 1} e^{\frac{2R-1}{2R^2}\chi}\,. \tag{4.2}$$

Assuming that $\frac{1}{2} < R < 1$ (which means that the corresponding compactification radius is between the Kosterlitz-Thouless and the self-dual one), the first equation for the sucseptability can also be rewritten in terms of scaling variables:

$$w = \mu\xi\,, \qquad \xi = \left(\lambda\sqrt{1/R - 1}\right)^{-\frac{2R}{2R-1}}, \tag{4.3}$$

where $\lambda = \sqrt{t_1 t_{-1}}$. The result reads

$$\begin{aligned} \chi &= R\log\xi + X(w)\,, \\ w &= e^{-\frac{1}{R}X} - e^{-\frac{1-R}{R^2}X}\,. \end{aligned} \tag{4.4}$$

5. Free energy and energy of the perturbed background

Let us consider the average over the Fermi sea of an observable $\mathcal{O}$ defined on the phase space

$$\langle \mathcal{O} \rangle = \frac{1}{2\pi} \iint\limits_{\text{Fermi sea}} dz_+ dz_-\, \mathcal{O}\left(z_+, z_-\right). \tag{5.1}$$

Here the boundary of the integration is defined by the profile equation (3.2). Since the change of variables described by eq. (3.3) is canonical (see eq. (3.5)), one can write

$$\langle \mathcal{O} \rangle = \frac{1}{2\pi} \int_{\mu} ds \int_{\omega_-(s)}^{\omega_+(s)} \frac{d\omega}{\omega}\, \mathcal{O}\left(z_+(\omega, s), z_-(\omega, s)\right). \tag{5.2}$$

The limits of integration over ω are defined by the cut-off. We choose it as two walls at $z_+ = \sqrt{\Lambda}$ and $z_- = \sqrt{\Lambda}$. Then the limits can be found from the equations

$$z_{\pm}(\omega_{\pm}(s), s) = \sqrt{\Lambda}\,. \tag{5.3}$$

First, consider the case of $\mathcal{O} = 1$. Then the integral (5.1) gives the number of fermions in the Fermi sea, i.e., $N =< 1 >$. Taking the derivative with respect to μ, we obtain

$$\partial_\mu N = -\frac{1}{2\pi} \log \frac{\omega_+(\mu)}{\omega_-(\mu)}\,. \tag{5.4}$$

In this case it is enough to restrict ourselves to the main order in the cut-off Λ for the boundary values $\omega_\pm$. From (5.3) and (3.3) we find to this order

$$\omega_\pm = \omega_{(0)}^{\pm 1}, \qquad \omega_{(0)} = \sqrt{\Lambda e^{\chi/R}}\,. \tag{5.5}$$

Combining (5.4) and (5.5), we find

$$\partial_\mu N = -\frac{1}{2\pi}\log\Lambda - \frac{1}{2\pi R}\chi\,. \tag{5.6}$$

Taking into account the relation (2.8), we find that the free energy coincides with the logarithm of the τ-function and $\beta = 2\pi R$ represents the time interval at which the theory is compactified

$$\mathcal{F} = -\frac{1}{\beta}\log\tau\,. \tag{5.7}$$

This derivation of the grand canonical free energy was done in [7]. However, using the general formula (5.2), we can also find the energy of the system, at least for the case of the Sine-Liouville theory. Since it is given by the integral of the free hamiltonian (2.6), from (5.2) and (4.1) one obtains

$$\partial_\mu E = -\partial_\mu\langle z_+ z_-\rangle = \frac{1}{2\pi}e^{-\frac{1}{R}\chi}\left\{(1+a_+a_-)\log\omega - Ra_+\omega^{-1/R} + Ra_-\omega^{1/R}\right\}\Big|_{\omega_-}^{\omega_+}\,. \tag{5.8}$$

To find the final expression one needs to know the limits $\omega_\pm$ in the second order in the cut-off. It is easy to show from (5.3) that eq. (5.5) should be changed by

$$\omega_\pm = \omega_{(0)}^{\pm 1}\left(1 \mp a_\pm \omega_{(0)}^{-1/R}\right)\,. \tag{5.9}$$

Substituting these limits into (5.8), one finds

$$\partial_\mu E = \frac{1}{2\pi}e^{-\frac{1}{R}\chi}\left\{(1+a_+a_-)\left(\log\Lambda + \frac{1}{R}\chi\right) - 2a_+a_-\right\} + \frac{R}{2\pi}(t_1+t_{-1})\Lambda^{1/2R}\,. \tag{5.10}$$

Integrating over μ and using $a_+a_- = \frac{R}{1-R}e^{\frac{2R-1}{R^2}X}$, we get

$$\begin{aligned}2\pi E = {} & \xi^{-2}\left(\frac{1}{2R}e^{-\frac{2}{R}X} + \frac{2R-1}{R(1-R)}e^{-\frac{1}{R^2}X} - \frac{1}{2(1-R)}e^{-2\frac{1-R}{R^2}X}\right)(\chi + R\log\Lambda) + \\ & +\xi^{-2}\left(\frac{1}{4}e^{-\frac{2}{R}X} - \frac{R}{1-R}e^{-\frac{1}{R^2}X} + \frac{R(4-5R)}{4(1-R)^2}e^{-2\frac{1-R}{R^2}X}\right) + R(t_1+t_{-1})\mu\Lambda^{1/2R}.\end{aligned} \tag{5.11}$$

We observe that the last term is non-universlal since it does not contain a singularity at $\mu = 0$. However, $\log\Lambda$ enters in a non-trivial way. It is combined with a non-trivial function of μ and ξ.

6. Thermodynamics of the system

We found above the energy of the Sine-Liouville theory (5.11) and also restored the free energy in the grand canonical ensemble for a general perturbation. In the Sine-Liouville case one can integrate the equation (4.4) to get the explicit expression [4]

$$2\pi\mathcal{F} = -\frac{1}{2R}\mu^2(\chi + R\log\Lambda) - \xi^{-2}\left(\frac{3}{4}e^{-\frac{2}{R}X} - \frac{R^2-R+1}{1-R}e^{-\frac{1}{R^2}X} + \frac{3R}{4(1-R)}e^{-2\frac{1-R}{R^2}X}\right). \tag{6.1}$$

Now we are going to establish thermodynamical properties of our system. It is expected to possess them since the spectrum of perturbations corresponds to the Matsubara frequencies typical for a system at a nonzero temperature. First of all, note that in the standard thermodynamical relations the free energy appears in the canonical, rather than in grand canonical ensemble. Therefore we define

$$F = \mathcal{F} - \mu\frac{\partial\mathcal{F}}{\partial\mu}. \tag{6.2}$$

It is easy to check that it can be written as

$$\begin{aligned} 2\pi F &= \frac{1}{R}\int^{\mu} s\chi(s)ds = \frac{1}{2R}\mu^2(\chi + R\log\Lambda) + \\ &+\xi^{-2}\left(\frac{1}{4}e^{-\frac{2}{R}X} - Re^{-\frac{1}{R^2}X} + \frac{R}{4(1-R)}e^{-2\frac{1-R}{R^2}X}\right). \end{aligned} \tag{6.3}$$

Following the standard thermodynamical relations, let us also introduce the entropy as a difference of the energy and the free energy:

$$S = \beta(E - F). \tag{6.4}$$

One finds from (5.11) and (6.3)

$$\begin{aligned} S &= \xi^{-2}\left(\frac{R}{1-R}e^{-\frac{1}{R^2}X} - \frac{1}{2(1-R)}e^{-2\frac{1-R}{R^2}X}\right)(\chi + R\log\Lambda) + \\ &+\xi^{-2}\left(-\frac{R^3}{1-R}e^{-\frac{1}{R^2}X} + \frac{R^2(3-4R)}{4(1-R)^2}e^{-2\frac{1-R}{R^2}X}\right) + \frac{R^2}{2\pi}(t_1 + t_{-1})\mu\Lambda^{1/2R}. \end{aligned} \tag{6.5}$$

Thus, all quantities are supplied by a thermodynamical interpretation. The role of the temperature is played by $T = \beta^{-1} = 1/2\pi R$. However, to have a consistent picture, the following (equivalent) thermodynamical relations should be satisfied:

$$\begin{aligned} S &= -\frac{\partial F}{\partial T} = 2\pi R^2\frac{\partial F}{\partial R}, \\ E &= \frac{\partial(\beta F)}{\partial\beta} = \frac{\partial(RF)}{\partial R}. \end{aligned} \tag{6.6}$$

In these relations it is important which parameters are hold to be fixed when we evaluate the derivatives with respect to R. Since we use the free energy in the canonical ensemble, one should fix the number of fermions N and the coupling constant λ but not the chemical potential μ.

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Another subtle point is that *a priori* the relation of our initial parameters to the corresponding parameters which are to be fixed may contain a dependence on the inverse temperature R. Therefore, in general we should redefine

$$\begin{aligned} \xi &\longrightarrow \xi = (a(R)\lambda)^{-\frac{2}{2-R}} \,, \\ \Lambda &\longrightarrow b(R)\Lambda \,, \\ F &\longrightarrow F + \frac{1}{2}c(R)\mu^2 \,. \end{aligned} \tag{6.7}$$

We find the functions $a(R)$, $b(R)$ and $c(R)$ from the condition (6.6), where N, λ and Λ are fixed. We emphasize that it looks rather non-trivial to us that it is possible to find such functions of R that this condition is satisfied. It can be considered as a remarkable coincidence indicating that the functions F, E and S do have the interpretation of thermodynamical quantities as free energy, energy and entropy correspondingly. In appendix A it is shown that the parameters should be taken as

$$a(R) = \left(\frac{1-R}{R^3}\right)^{1/2} \,, \qquad b(R) = 1 \,, \qquad c(R) = 0 \,. \tag{6.8}$$

It is worth noting several features of this solution. First of all, the plausible property is that we do not redefine the cut-off and the free energy. On the other hand, the constant λ coincides with its initial definition (4.3) if to absorb one R factor into definition of the coupling constants [2] $t_{\pm 1}$ what probably indicates the right relation of λ with the corresponding coupling constant in the conformal Sine-Liouville theory. Another important consequence of the choice (6.8) is that the entropy (6.5) is proportional to λ^2 and therefore vanishes in the absence of perturbations. It appears to be non-zero only when the background of the string theory is described by a time dependent Fermi sea. However, a puzzle still remains how to calculate the entropy in a similar way as the energy was evaluated, that is by directly identifying the corresponding degrees of freedom which give rise to the same macroscopic state and, hence, the entropy.

6.1 The "black hole" limit

Let us restrict ourselves to the limit $\mu \to 0$. If we perturbed the 2D string theory by the vortex, instead of the tachyon, modes it would correspond to the black hole limit [3, 4]. In this limit we have $X = 0$. As a result, we obtain from (6.3), (5.11) and (6.5) for all three thermodynamical quantities:

$$\begin{aligned} 2\pi F &= \frac{(2R-1)^2}{4(1-R)}\tilde{\lambda}^{\frac{4R}{2R-1}}, \\ 2\pi E &= \frac{2R-1}{2R(1-R)}\left(\log(\Lambda\xi) - \frac{R}{2(1-R)}\right)\tilde{\lambda}^{\frac{4R}{2R-1}}, \\ S &= \frac{2R-1}{2(1-R)}\left(\log(\Lambda\xi) - \frac{R^2(3-2R)}{2(1-R)}\right)\tilde{\lambda}^{\frac{4R}{2R-1}}, \end{aligned} \tag{6.9}$$

[2] This corresponds to the fact that one should not introduce such R factor in front of the perturbing potential as was done in [4].

where $\tilde{\lambda} = a(R)\lambda$. It is worth to note an interesting feature of this solution. Since $\log(\Lambda\xi) \gg 1$, we get $2\pi E \approx S$ and in comparison with these quantities F is negligible. This is to be compared with the result of [19] that the analysis of the dilatonic gravity derived from 2D string theory leads to the vanishing free energy and to the equal energy and entropy. We see that it can be true only in a limit, but the free energy can never be exactly zero since it is its derivative with respect to the temperature that produces all other thermodynamical quantities.

For the special case of $R = 2/3$ which is T-dual to the stringy black hole model, we find:

$$\begin{aligned} \beta F &= \frac{1}{18}\tilde{\lambda}^8 \,, \\ \beta E &= \left(\frac{1}{2}\log(\Lambda\xi) - \frac{1}{2}\right)\tilde{\lambda}^8 \,, \\ S &= \left(\frac{1}{2}\log(\Lambda\xi) - \frac{5}{9}\right)\tilde{\lambda}^8 \,. \end{aligned} \tag{6.10}$$

For the dual compactification radius $R = 3/2$, all quantities change their sign:

$$\begin{aligned} \beta F &= -3\tilde{\lambda}^3 \,, \\ \beta E &= -\left(2\log(\Lambda\xi) - 3\right)\tilde{\lambda}^3 \,, \\ S &= -2\log(\Lambda\xi)\tilde{\lambda}^3 \,. \end{aligned} \tag{6.11}$$

This might be interpreted as a result for the background dual to the black hole of a negative mass, which does appear in the analysis of [20].

7. Conclusions and problems

The main puzzle of the black hole physics is to find the states corresponding to the entropy of a classical gravitating system. In our case, instead of a black hole, the role of such system is played by a nontrivial tachyonic background. However, it manifests the similar properties as the usual dilatonic black hole, which appears as a T-dual perturbation of the initial flat spacetime. Therefore, it is not surprising that the studied background also possesses a nonvanishing entropy.

In our case we have a little more hope to track the origin of the entropy, than, for example, in the study of the Schwarzschild black hole, since we are dealing with the well defined string theory and can identify its basic degrees of freedom. It allowed us in this paper to find microscopically the energy and free energy of the "Sine-Liouville" background. The entropy was calculated not independently, but followed from the basic thermodynamical relations. Our main result is the coincidence of this microscopic and the thermodynamical energies. The latter is calculated as a derivative of the free energy with respect to the temperature. Note that the "temperature" appears from the very beginning as a parameter of the tachyonic perturbation, and not as a macroscopic characteristic of a heated system. That is why the thermodynamical interpretation comes as a surprise.

An analogous situation can be seen in the Unruh effect [21]. There the temperature seen by the accelerating observer is defined by the value of the acceleration, which does not *a priory* have any relation to thermodynamics. In fact, in the latter case the temperature can be found from the analysis of the quantum field theory on the Rindler space which is the spacetime seen by the accelerating observer. The same should be true for our case. It should be possible to reproduce the global structure of the tachyonic background from the matrix model solution, and then the thermodynamical interpretation should appear from the analysis of the spectrum of particles detected by some natural observer [22].

One might try to find a hint to the thermodynamical behaviour of our system in the very similar results for the T-dual system of [4] where instead of tachyons we have the perturbation by vortices. There the euclidean time is compactified from the very beginning and the Gibbs ensemble is thus explicitly introduced with R as a temperature parameter. On the other hand, the coincidence of microscopic energies in two mutually dual formulations (up to duality changes in parameters, like $R \to 1/R$) is far from obvious. Even how to find the energy in this dual case is not clear.

In conclusion, we hope that our observation will help to find a microscopic approach to the calculation of entropy at least in the 2D string theory with nontrivial backgrounds. Together with the plausible conjecture that the dual system describes the 2D dilatonic black hole, it could open a way for solving the puzzle of the microscopic origin of entropy in the black hole physics.

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A. Calculation of $\partial F/\partial R$

In this appendix we calculate the derivative of the free energy (6.3) with respect to R. First, note that due to (6.2) and (2.8), we have

$$\left(\frac{\partial F}{\partial R}\right)_{N,\lambda,\Lambda} = \left(\frac{\partial \mathcal{F}}{\partial R}\right)_{\mu,\lambda,\Lambda} = \left(\frac{\partial \mathcal{F}}{\partial R}\right)_{\mu,\xi,\Lambda} + \left(\frac{\partial \xi}{\partial R}\right)_{\lambda} \left(\frac{\partial \mathcal{F}}{\partial \xi}\right)_{\mu,\Lambda,R}. \tag{A.1}$$

From (6.1) and (6.7) one can obtain

$$2\pi\xi \left(\frac{\partial \mathcal{F}}{\partial \xi}\right)_{\mu,\Lambda,R} = -\xi^{-2}\frac{R(2R-1)}{1-R}\left(e^{-\frac{1}{R^2}X} - \frac{1}{2R}e^{-2\frac{1-R}{R^2}X}\right), \tag{A.2}$$

$$\xi^{-1}\left(\frac{\partial \xi}{\partial R}\right)_{\lambda} = -\frac{1}{R(2R-1)}\left(\log\xi + 2R^2\frac{d\log a(R)}{dR}\right). \tag{A.3}$$

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The most complicated contribution comes from the first term in (A.1). From (6.1), (6.7) and (4.4) one finds

$$
\begin{aligned}
2\pi \left(\frac{\partial \mathcal{F}}{\partial R}\right)_{\mu,\xi,\Lambda} &= \xi^{-2}\left(\frac{1}{1-R}e^{-\frac{1}{R^2}X} - \frac{1}{2R(1-R)}e^{-2\frac{1-R}{R^2}X}\right)\frac{X}{R} + \\
&+\xi^{-2}\left(\frac{R(2-R)}{(1-R)^2}e^{-\frac{1}{R^2}X} - \frac{3}{4(1-R)^2}e^{-2\frac{1-R}{R^2}X}\right) - \\
&-\frac{1}{2}\mu^2\left(\frac{d\log b(R)}{dR} + \frac{dc(R)}{dR}\right). \qquad \text{(A.4)}
\end{aligned}
$$

Putting the results (A.2), (A.3) and (A.4) together, we obtain

$$
\begin{aligned}
2\pi \left(\frac{\partial F}{\partial R}\right)_{N,\lambda,\Lambda} &= \frac{1}{R^2}\xi^{-2}\left(\frac{R}{1-R}e^{-\frac{1}{R^2}X} - \frac{1}{2(1-R)}e^{-2\frac{1-R}{R^2}X}\right)\chi + \\
&+\xi^{-2}\left(\frac{R(2-R)}{(1-R)^2}e^{-\frac{1}{R^2}X} - \frac{3}{4(1-R)^2}e^{-2\frac{1-R}{R^2}X}\right) + \\
&+\xi^{-2}\left(\frac{2R^2}{1-R}e^{-\frac{1}{R^2}X} - \frac{R}{1-R}e^{-2\frac{1-R}{R^2}X}\right)\frac{d\log a(R)}{dR} - \\
&-\frac{1}{2}\mu^2\left(\frac{d\log b(R)}{dR} + \frac{dc(R)}{dR}\right). \qquad \text{(A.5)}
\end{aligned}
$$

We observe that the term proportional to the susceptibility χ coincides with the corresponding term in the entropy (6.5) what can be considered as the main miracle of the present derivation. To get the coincidence of other terms, we obtain a system of three equations, from which only two are independent:

$$
\begin{aligned}
\frac{d\log b(R)}{dR} + \frac{dc(R)}{dR} &= 0\,, \\
2\frac{d\log a(R)}{dR} - \frac{1}{R^2}\log b(R) &= \frac{2R-3}{R(1-R)}\,. \qquad \text{(A.6)}
\end{aligned}
$$

Since we have only two equations on three functions, there is an ambiguity in the solution. It can be fixed if we require that the entropy vanishes without perturbations. Then we come to the following result

$$
a(R) = \left(\frac{1-R}{R^3}\right)^{1/2}, \qquad \log b(R) = 0\,, \qquad c(R) = 0\,, \qquad \text{(A.7)}
$$

which ensures the thermodynamical relations (6.6). Note that in fact they are fulfilled only up to terms depending on the cut-off $\log\Lambda$. Such terms can not be reproduced by differentiating with respect to the temperature, but they are considered as non-universal in our approach and thus can be dropped.

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